PDF Error Discussion:
the sequel to the sequel

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Global pdf fits

- Calculation of production cross sections at the Tevatron relies upon knowledge of pdfs in relevant kinematic range
- pdfs are determined by global analyses of data from DIS, DY and jet production
- Two major groups that provide semi-regular updates to parton distributions when new data/theory becomes available
  - MRS->MRST98->MRST99->MRST2001->MRST2002
  - CTEQ->CTEQ5->CTEQ5(1)->CTEQ6->CTEQ6.1
  - also GKK and Alekhin, but not widely used
- All of the above groups provide a way to estimate the error on the central pdf
  - new methodology enables full characterization of parton parametrization space in neighborhood of global minimum
    - Hessian method
    - Largrange Multiplier
  - both of above techniques used by CTEQ and MRST

Nuts/bolts of fits

- Functional form used in CTEQ fits is:
  - $xf(x, Q_o) = A_o\, x^{A1}\, (1-x)^{A2}\, e^{A3x}\, (1 + A_4 x)^{A5}$
    - $Q_o = 1.3$ GeV (below any data used in fit)
      - easier to do forward evolution than backward
      - MRST starts at 1 GeV (- gluon distribution)
    - functional form arrived at by adding a 1:1 Pade expansion to quantity d(log xf)/dx
    - more versatile than form used in CTEQ5 or MRST
    - there are 20 free parameters used in the global fit
      - MRST has 15 free parameters
- Light quarks treated as massless; evolution kernels of PDFs are mass-independent
- Zero mass Wilson coefficients used in DIS structure functions
- NB: MRST pdf's not in pure MS-bar scheme; use Roberts-Thorne treatment of heavy quarks at threshold
  - maybe noticeable only at low x

Uncertainties in pdf fits

- ● Two sources
  - ◆ Experimental errors
    - ▲ Hessian/Lagrange multiplier techniques designed to address estimate of these effects
      - – question is what $\Delta\chi^2$ change best represents estimate of uncertainty (CTEQ uses $\Delta\chi^2$ of 100 (out of 2000) for 90% CL limit; MRST uses $\Delta\chi^2$ of 50); GKK/Alekhin uses 1 (for 1 sigma error)
      - – for details on the choice of $\Delta\chi^2$, see the presentation on 2/27/03
  - ◆ Theoretical
    - ▲ higher twist/non-perturbative effects
      - – choose Q^2 and W cuts to try to avoid
    - ▲ higher order effects
      - – is NNLO necessary yet?
    - ▲ edge of phase space effects
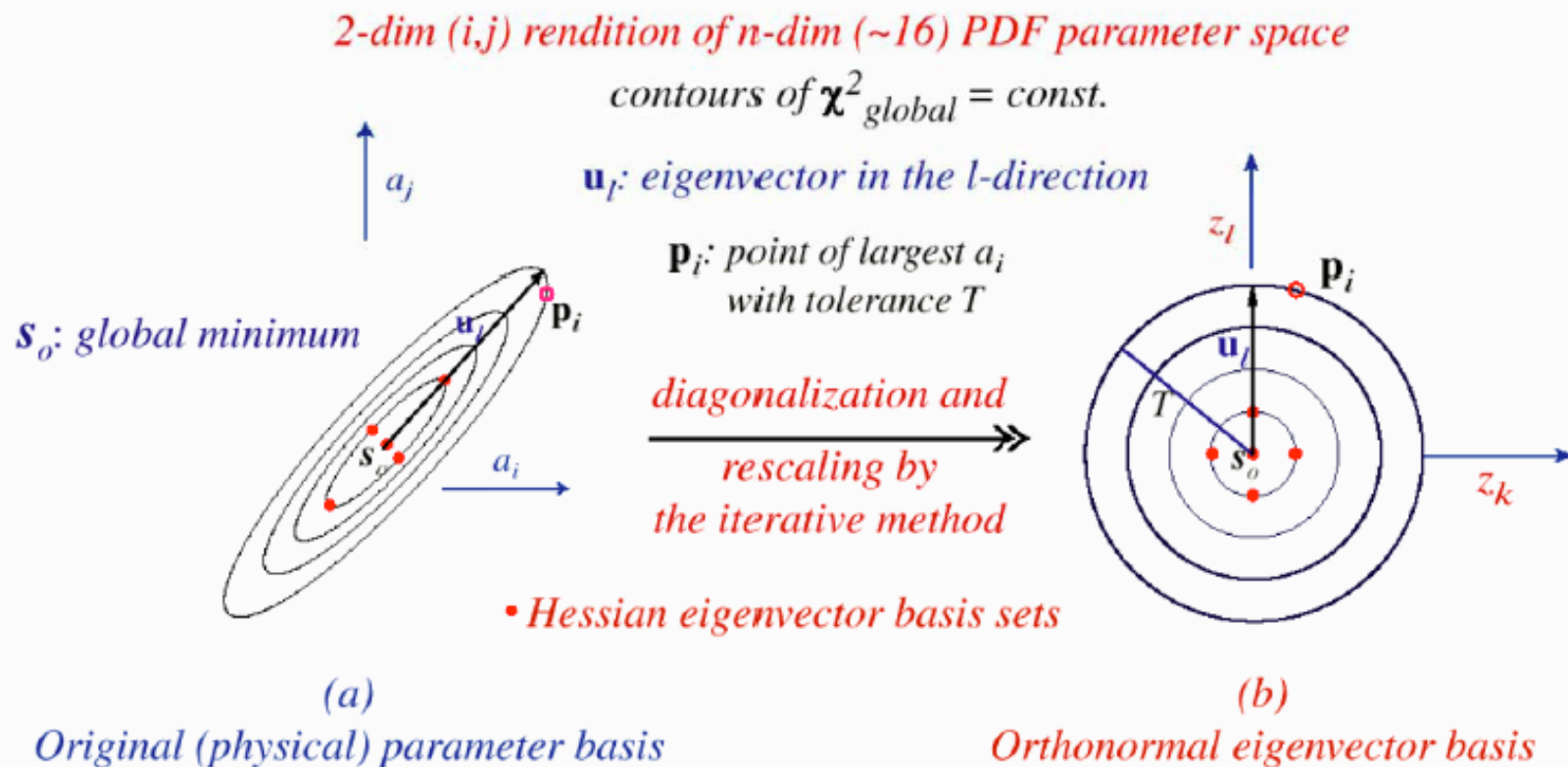   - – threshold resummation needed?
    - ▲ note that for the most part, CTEQ and MRST make the same cuts/assumptions so theoretical *precision* should be better than theoretical *accuracy*

Hessian method

More accessible to experimenters than LM technique.

The Hessian Method of quantifying uncertainties by a complete set of orthonormal eigenvector PDFs


2-dim (i,j) rendition of n-dim (~16) PDF parameter space

contours of χ^2_{global} = const.

u_l: eigenvector in the l-direction

p_i: point of largest a_i with tolerance T

s_o: global minimum

diagonalization and rescaling by the iterative method

• Hessian eigenvector basis sets

(a)
Original (physical) parameter basis

(b)
Orthonormal eigenvector basis

Discussion

- Before we go on, let's distinguish between the pdf uncertainty on a cross section and the pdf uncertainty on an acceptance

- The latter affects our reported results and can only be done by us; the former is just quoted when comparing our results to theory and can be done by anyone

Using pdf uncertainties

- CTEQ or MRST pdf's are meant to be used with NLO programs

- It's relatively straightforward, for example, to calculate the pdf uncertainty for something like the t-tbar total cross section using a NLO calculation

includes effects of varying α_s
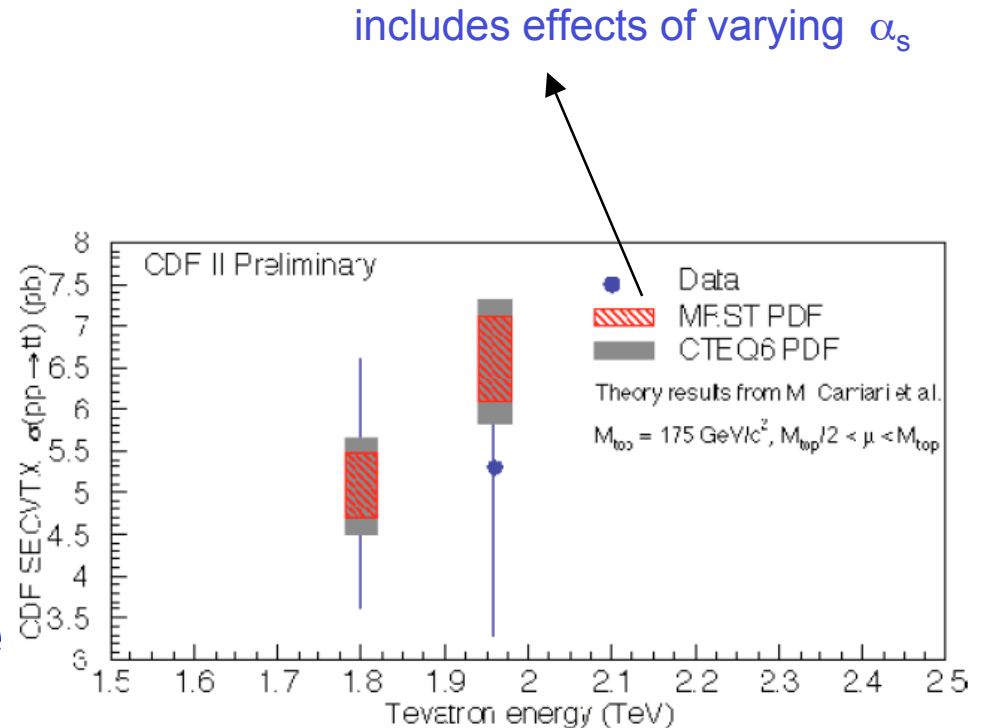
Example: uncertainties for Run 2 jet cross sections

20 eigenvectors, 2 directions, so 40 error pdf's

- CDF will measure the inclusive jet cross section in the forward regions as well
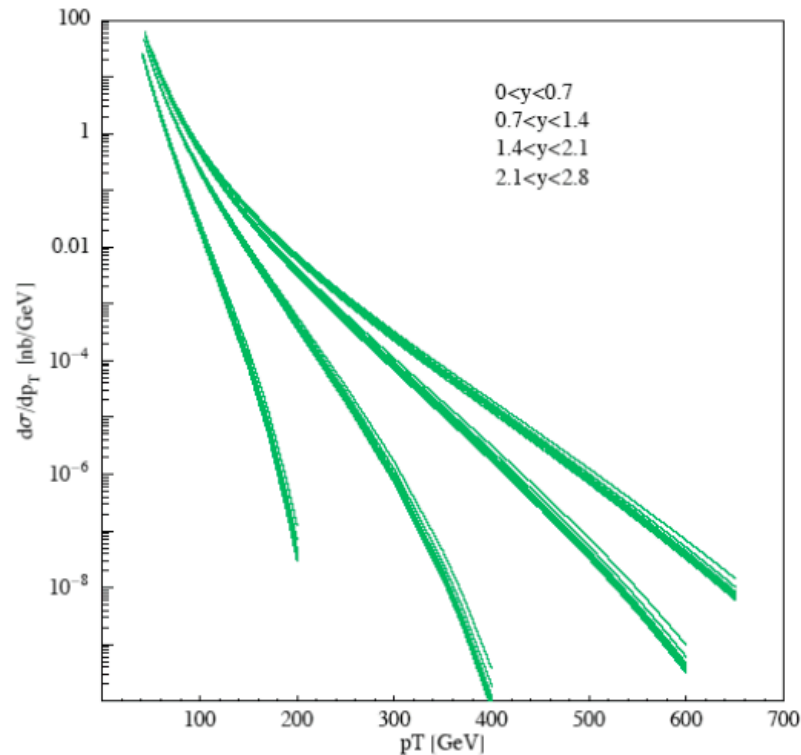
one eigenvector (15) provides the extremes for the jet cross section
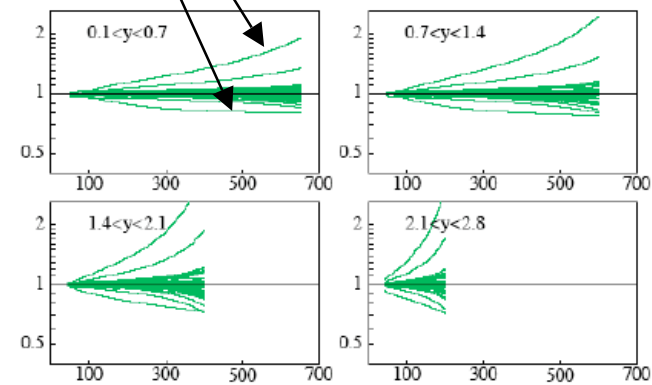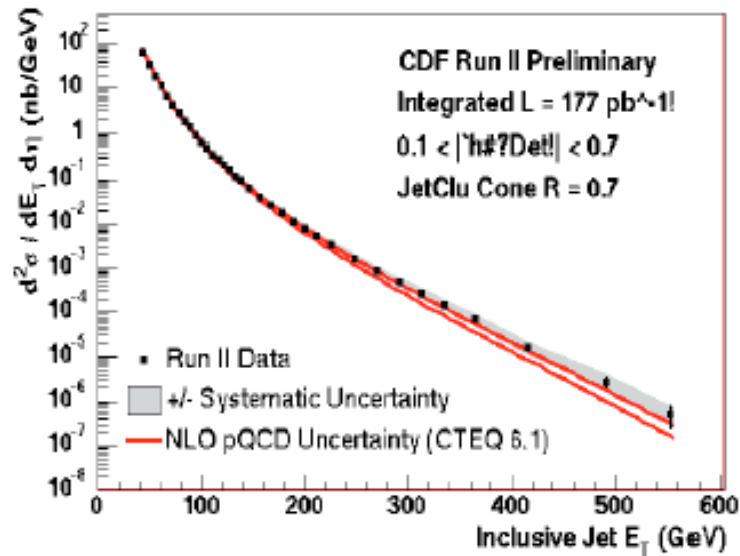

0<y<0.7
0.7<y<1.4
1.4<y<2.1
2.1<y<2.8

Figure 28: Uncertainty range of the Run 2 cross section for the CDF rapidity bins. The curves show the ratios of the 40 eigenvector basis sets compared to the central (CTEQ6.1M) prediction

NB: new physics is primarily central; a pdf explanation should work everywhere
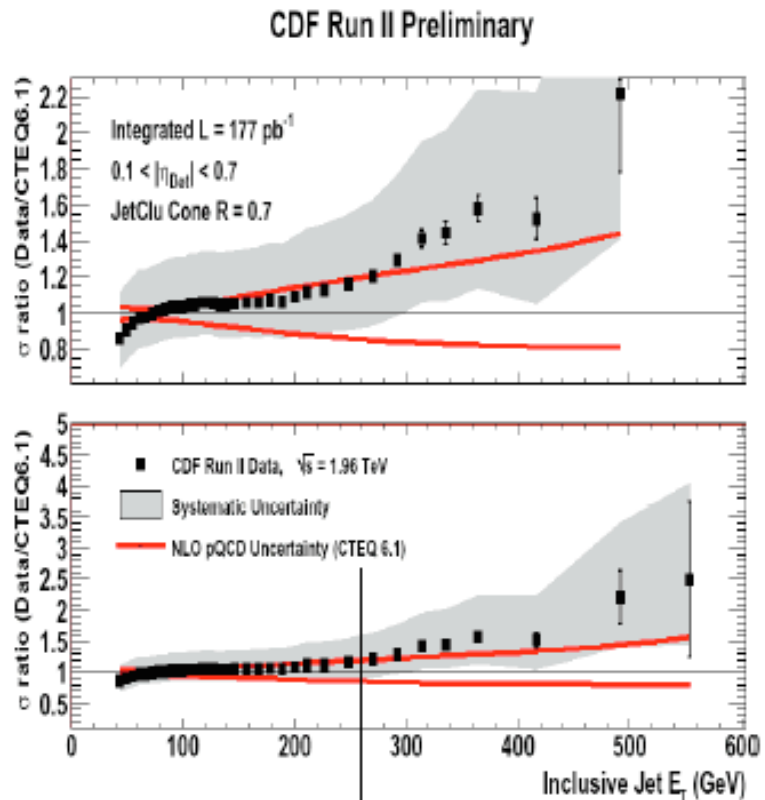
CDF Run 2 jet cross sections

Jet cross section in agreement with theoretical prediction within errors

Jet energy scale error will decrease as understanding of Run 2 detector improves

Dominant theoretical error is due to pdf uncertainty

CTEQ6.1 already has an enhanced high x gluon due to influence of Run 1 jet data
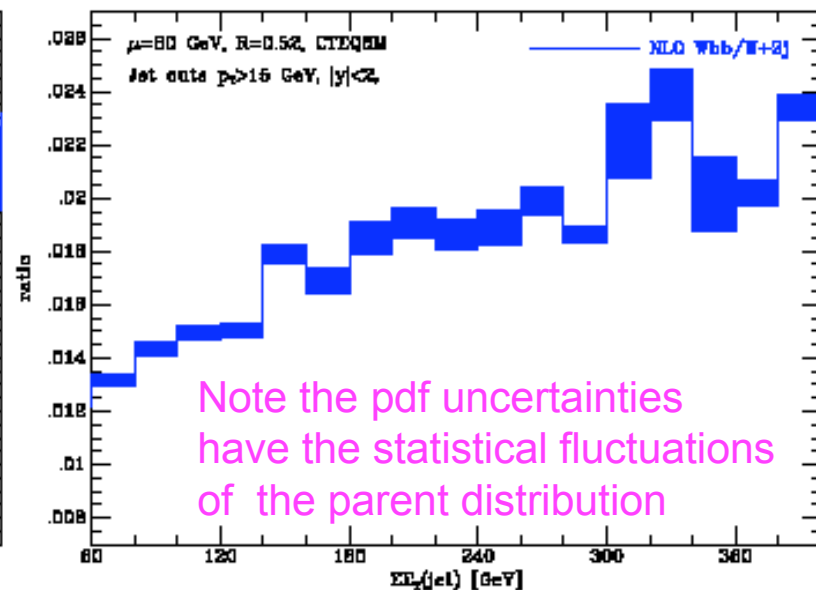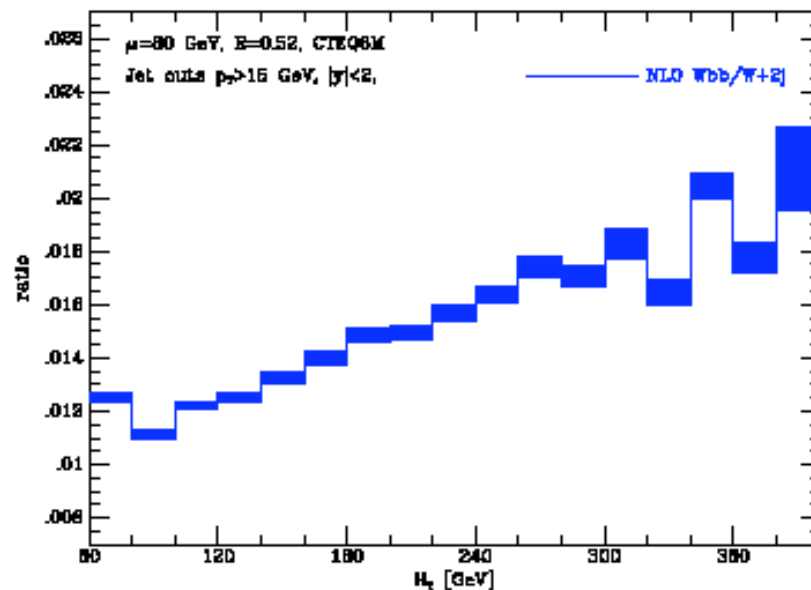
Using pdf uncertainties

NLO programs can be slow, especially if you have to run 41 pdf's
But if new version of LHAPDF is used, can run full cross section with central pdf and store
pdf*pdf luminosity for each event and then re-weight

■ Total cross-section uncertainty: Using MCFM, see CDF6849

$$W b\bar{b} \rightarrow 2.5\% , \qquad W + 2j \rightarrow 1.5\%.$$

■ Uncertainty in the $(W b\bar{b}/W + 2 \text{ jet})$ ratio:

Note the pdf uncertainties
have the statistical fluctuations
of the parent distribution

Using pdf uncertainties

- More often, though, we need to calculate the pdf uncertainty on something like an acceptance

- And we often use a Monte Carlo so that we can work at the detector level

- Technically, should use LO pdf's with MC's
  - no LO error pdf's
  - but resulting error of using NLO pdf's is NLO, beyond scope of MC

- But, LO central fit can differ from NLO central fit by more than NLO pdf uncertainty

- Also, have to worry about generating enough Monte Carlo events to reduce statistical errors

$$\pm\frac{1}{2}\sqrt{\sum_{i=1}^{15}(\sigma(2i-1)-\sigma(2i))^2}$$
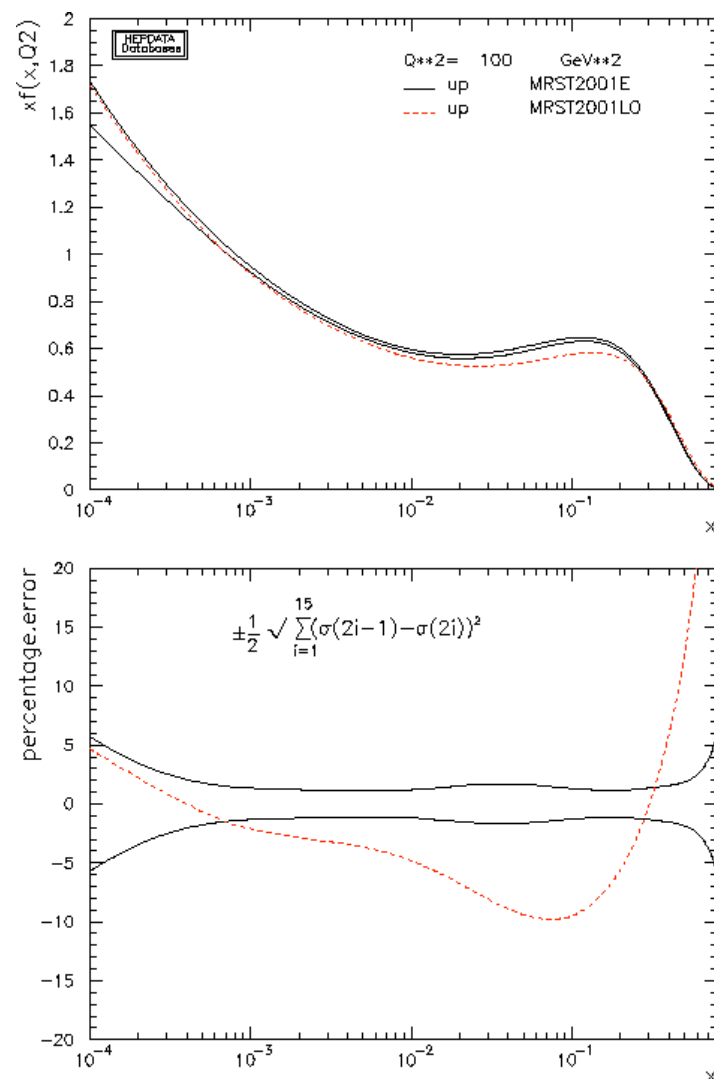
Uncertainties on cross sections

● $\sigma(t\bar{t}) = f_1(x_1,M) \otimes f_2(x_2,M) \otimes \sigma(\alpha_s(\mu))$

◆ $\Delta\sigma \to \Delta f_1, \Delta f_2, \Delta\alpha_s, (\Delta(\mu,M))$

◆ can use CTEQ/MRST Hessian pdf error sets to estimate pdf uncertainty

▲ NB: $\Delta\sigma$ is really a NLO quantity

● Use MCFM

◆ easy to integrate long enough so that the statistical error is insignificant

◆ can use pdf weights to speed up calculation

◆ can compare LO,NLO

◆ gives John more citations

◆ why use a parton shower Monte Carlo if what you want to do is to evaluate a matrix element?

Use NLO and LO ME for t-tbar production

- **Run t-tbar in MCFM**
  - ◆ total cross section at NLO
  - ◆ lepton + jets and dilepton at LO using CDF cuts
    - ▲ NLO using cuts available in the future
    - ▲ since error pdf sets are all at NLO, use NLO pdf's for both LO and NLO calculations

- For full cross section at NLO, find cross section uncertainty of 6.0 +0.54/-0.43 pb (5.915 +0.21/-0.14) with CTEQ (MRST) pdf's
  - ◆ +9.1%, -7.2%
  - ◆ +3.8%,-2.4%
  - ◆ CTEQ/MRST σ's almost exactly the same
  - ◆ different uncertainties due to different criteria as to range of allowable fits
- For full cross section at LO, find cross section uncertainty of 5.34 + 0.42/-0.35 (5.29 +0.16/-0.1)
  - ◆ +7.9%, -6.6%
  - ◆ +3.0%,-1.9%

NLO vs LO

- Why is fractional $\Delta\sigma$ larger at NLO than at LO?
- Break down by subprocess and order on right (for CTEQ/MRST pdf's)
- K-factor for gg is >> than K-factor for qq
- Uncertainty for cross section for gg initial state is >> than that for qq initial state
- NLO $\Delta\sigma$ > LO $\Delta\sigma$
 - Nota bene

- LO:
  - qq   4.76(+0.16,-0.20)
  - qq   4.82(+0.05,-0.04)
  - gg   0.58(+0.29,-0.21)
  - gg   0.47(+0.08,-0.07)
- NLO
  - qq   5.12(+0.12,-0.13)
  - gg   0.92(+0.41,-0.31)
  - gq   -0.07(+0.01,0.01)

Also, NB

- NLO gluon is higher than LO gluon at high x so if you use a LO program like Herwig/Pythia and a LO pdf then gg contribution is going to be much smaller than using a NLO program with a NLO pdf

- Guess: expect gg contribution to give a lower acceptance than qq

  - x_1 and x_2 for gg initial state have a wider spread than for qq

  - more boosts so more jets lost because of rapidity cuts

  - ISR causes an effect in the opposite direction

    - how large?
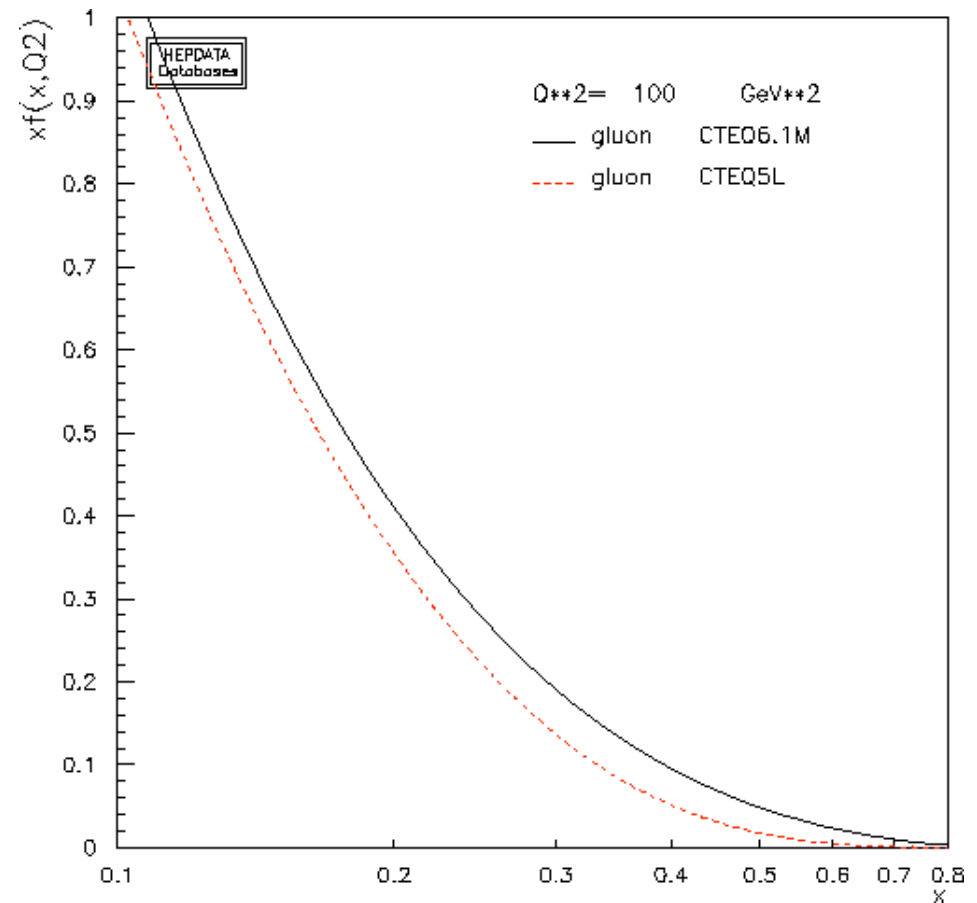
Now look at final states

- **Uncertainty in $l\nu b\bar{b}jj$, $l^+l^-\nu\nu b\bar{b}$ final states same as tt until you make cuts**

- **Now apply cuts shown on the right**

- **Use LO MCFM**

- **For brevity, top results shown here are for lepton+jets final state; also have results for dilepton**

- **lepton+jets cuts (W+ only)**
  - $p_T^{electron} > 20$ GeV
  - $|\eta^{electron}| < 1.1$
  - $p_T^{neutrino} > 20$ GeV
  - $R_{cone} = 0.4$
  - $E_T^{jet} > 15$ GeV/c; $|\eta^{jet}| < 2.0$

- **dilepton cuts**
  - lepton E_T: > 20 GeV
  - electrons: eta < 2
  - muons: eta ~< 1.1
  - jets: > 15 GeV eta < 2.5
  - MET: > 25 GeV
 - HT (leptons + MET + all jets): > 200 GeV

Lepton + jets final state

- **4 jet final state**
  - CTEQ/MRST
- **Central cross section**
  - 112.6 fb
  - 112.3 fb
- **Minimum cross section**
  - 109.2 fb (pdf 29)   **-3%**
  - 111.1 fb (pdf 19)   **-1.1%**
- **Maximum cross section**
  - 118 fb (pdf 30)   **+4.8%**
  - 114 fb (pdf 18)   **+1.4%**

- **3 or 4 jet final state**
  - CTEQ/MRST
- **Central cross section**
  - 203.9 fb
- **Minimum cross section**
  - 197.6 fb (pdf 29) **-3.1%**
- **Maximum cross section**
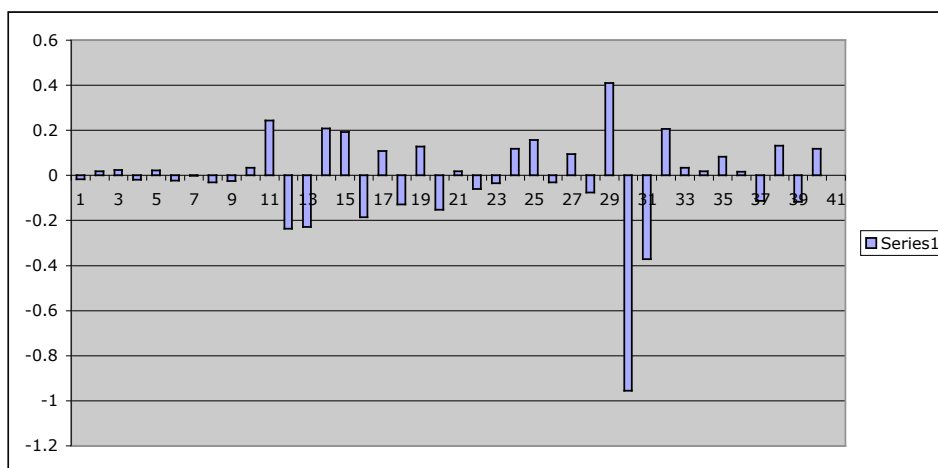 - 214.2 fb (pdf 30) **+5%**

Acceptance uncertainty:CTEQ

- **Why would acceptance vary with changes in pdf's/α_s?**
  - ◆ effects of rapidity cuts
    - ▲ boosts along z-axis
  - ◆ effects of jet E_T cuts
- **Expect acceptance variation to be a 2nd order effect, i.e. much smaller than $\Delta\sigma$**
- **ΔA is << $\Delta\sigma$**
- **Note that pdf 29 has smallest gg component and has largest acceptance; pdf 30 has largest acceptance and pdf 29 has lowest acceptance**
- **Note also that there is very little variation in acceptance for the larger (quark-dominated) eigenvectors**

LO 4-jet cross section after cuts
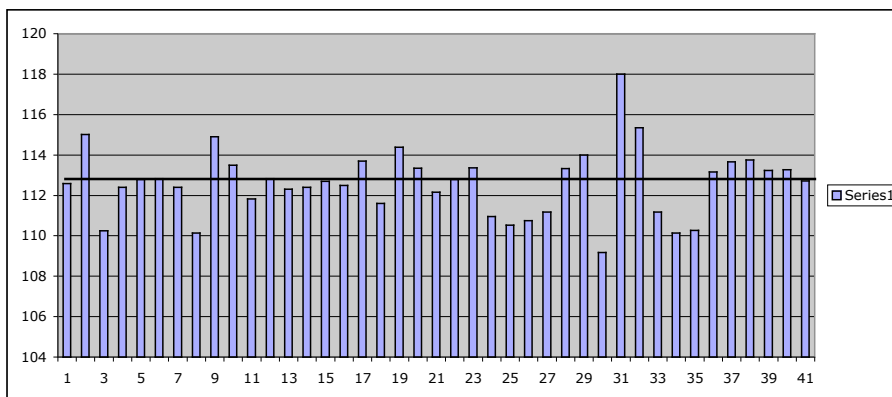
Variation in acceptance (%) for 4-jet

Acceptance uncertainty:MRST

- Why would acceptance vary with changes in pdf's/α_s?
  - effects of rapidity cuts
    - ▲ boosts along z-axis
  - effects of jet E_T cuts
- Expect acceptance variation to be a 2nd order effect, i.e. much smaller than $\Delta\sigma$
- ΔA is $<< \Delta\sigma$
- Note that scale is very different than for CTEQ plot
- Note also:
  - no dominant eigenvector
  - more + excursions than −

LO 4-jet cross section after cuts
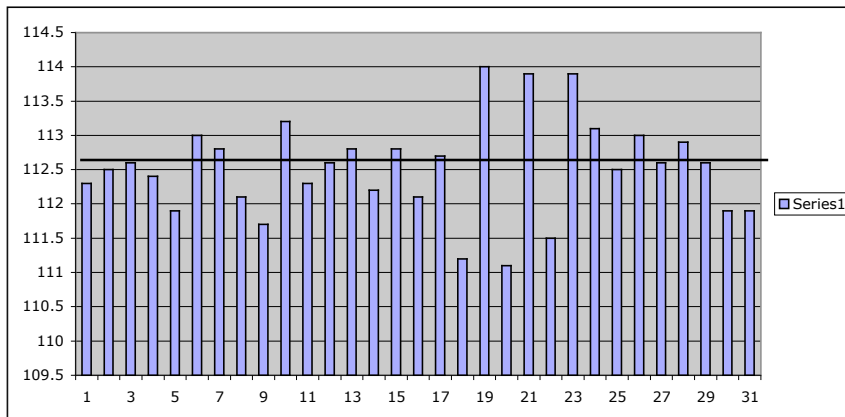
Variation in acceptance (%) for 4-jet
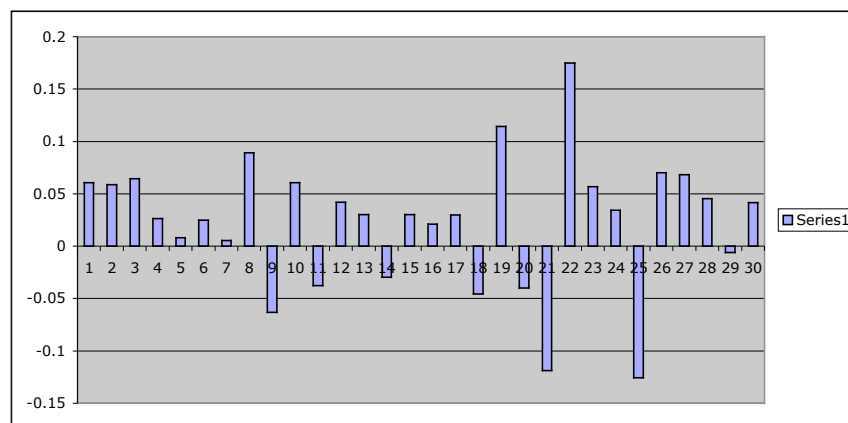
Acceptance uncertainties

- 3 or 4 jet final state
  - CTEQ
- Central acceptance
  - 0.515
- Minimum acceptance
  - 0.512 (pdf 30) **-0.7%**
- Maximum acceptance
  - 0.517 (pdf 29) **+0.4%**

- 4 jet final state
  - CTEQ/MRST
- Central acceptance
  - 0.285
  - 0.2870
- Minimum acceptance
  - 0.282 (pdf 30)   **-1%**
  - total negative acceptance uncertainty   **-1.1%**
  - .2867 (pdf 26)   **-0.13%**
  - total negative uncertainty   **-0.2%**
- Maximum acceptance
  - 0.286 (pdf 29)   **+0.4%**
  - total positive acceptance uncertainty   **+0.75%**
  - 0.2874 (pdf 22)   **+0.18%**
  - total positive acceptance uncertainty   **+0.3%**

Some other tests

- **LO vs NLO**
  - ◆ CTEQ6L1
    - ▲ σ_{4jet}(LO)=442 fb
    - ▲ σ_{4jet}(cuts)=125 fb
    - ▲ A=0.283
- **Varying α_s**
  - ◆ MRST2001(α_s=.117 (default=.119))
    - ▲ σ_{4jet}(LO)=377 fb
    - ▲ σ_{4jet}(cuts)=108 fb
    - ▲ A=0.286
  - ◆ MRST2001(α_s=.121)
    - ▲ σ_{4jet}(LO)=397 fb
    - ▲ σ_{4jet}(cuts)=114 fb
    - ▲ A=0.287

- **Reminder**
  - ◆ A_{MRST}=0.287
  - ◆ A_{cteq6}=0.285
  - ◆ A_{29}=0.286
  - ◆ A_{30}=0.282
- **So?**
  - ◆ we're happy because changing order, α_s and pdf group causes changes in the acceptance of same order or less as error pdf's

Summary

- ## So what do we know?
  - ◆ pdf uncertainties for LO cross sections on the order of 5%
  - ◆ pdf uncertainties on LO acceptances on the order of 1%
    - ▲ CTEQ and MRST central fits give similar answers
    - ▲ LO pdf's give similar answer for acceptance as NLO
    - ▲ varying α_s has reasonably small effect on acceptance
- ## What don't we know?
  - ◆ pdf uncertainties on NLO acceptances
  - ◆ combined effects of pdf uncertainties and ISR (at NLO)
  - ◆ additional effects when detector simulation added

Addressing the latter

- Look at t-tbar with **MC@NLO**
  - know that $\Delta\sigma$ is greater at NLO than at LO
  - what about ΔA? more gg
    - ▲ compare CTEQ6.1,CTEQ6L1,pdf's 29 and 30
  - no spin correlations; run Herwig w/wo spin correlations to see effect
- **MC@NLO** also adds new variables
  - additional radiation
    - ▲ important consideration for top mass (use 29 and 30 to estimate)
    - ▲ increase in acceptance due to initial state radiation satisfying jet cuts (especially important for gg)
    - ▲ lepton obliteration
  - hadronization
    - ▲ jet degradation
  - detector simulation
    - ▲ jet degradation

Summarize all in a CDF note